पुणा International School
Shree Swaminarayan Gurukul, Zundal

# Grade – 10 maths Specimen copy Year 21-22

# INDEX

**CHAPTER NO. – 1**

**CHAPTER NAME – REAL NUMBERS**

## KEY POINTS TO REMEMBER –

- **Natural Numbers: Counting Numbers are called Natural Numbers are denoted by**

  **N = {1, 2, 3, 4, 5, …….}**

- **Whole Numbers : The collection of Natural Numbers along with zero is the collection of Whole Numbers and is denoted by W.**

  **W = {0, 1, 2, 3, 4, …..}**

- **Integers: The collection of Natural numbers, their negatives along with the number zero are called Integers. This collection is denoted by Z.**

  **Z = {……..-3, -2, -1, 0, 1, 2, 3……}**

- **Rational number: The numbers, which are obtained by dividing two integers, are called Rational numbers. Division by zero is not defined.**

  **Q = { p/q: p and q are integers ,q ≠0}**

- **Prime number: The number other than 1, with only factors namely 1 and the number itself, is a prime number.**

  **{ 2, 3, 5, 7, 11, 13, 17, 19,……..}**

- **Co-prime number: If HCF of two numbers is 1, then the two numbers are called co-prime.**

### Euclid's division lemma :

- For given positive integers 'a' and 'b' there exist unique whole numbers 'q' and 'r' satisfying the relation $a = bq + r$, $0 \leqslant r < b$.

  **Theorem:** If a and b are non-zero integers, the least positive integer which is expressible as a linear combination of a and b is the HCF of a and b,

  i.e. if d is the HCF of a and b, then these exist integers $x_1$ and $y_1$,

  such that $d = ax_1 + by_1$ and d is the smallest positive integer which is expressible in this form.

The HCF of a and b is denoted by HCF( a, b)

**. Euclid's division algorithms :**

- HCF of any two positive integers a and b. With $a > b$ is obtained as follows:

  **Step 1 :** Apply Euclid's division lemma to a and b to find q and r such that

  $a = b q + r, \ 0 \leqslant r < b.$

  b = Divisor

  q = Quotient

  r = Remainder

  **Step II**: If $r = 0$, HCF (a ,b) = b if $r \neq 0$, apply Euclid's lemma to b and r.

  **Step III:** Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

- **The Fundamental Theorem of Arithmetic**

  **Every composite number can be expressed (factorized) as a product of primes and this factorization is unique, apart from the order in which the prime factors occur.**

  **Ex : 24 = 2 X 2 X 2 X 3 = 3 X 2 X 2 X 2**

  **The Fundamental Theorem of Arithmetic says that every composite number can be factorized as a product of primes.**

- **HCF and LCM:( by prime factorization method)**

**HCF: Product of the smallest power of each common prime factor in the numbers.**

**LCM: Product of the greatest power of each common prime factor in the numbers.**

- **For any two positive integers a and b**

  **HCF (a x b) X LCM (a x b) = a x b**

- **Revisiting Irrational Numbers**

**Theorem 1.3: Let p be a prime number. If p divides $a^2$, then p divides a, Where a is a positive integer.**

**Theorem1.4: $\sqrt{2}$ is irrational.**

- **Revisiting Rational Numbers and Their Decimal Expansions**

**Theorem 1.5 : Let $x$ be a rational number. Whose decimal expansion terminates then x can be expressed in the form $\frac{p}{q}$ . Where p and q are co-prime, and prime factorization of q is of the form $2^m 5^n$, where m, n are non negative integers.**

**Theorem 1.6: Let $x = \frac{p}{q}$, q $\neq$ 0 to be a rational number, such that the prime factorization of q is not of the form $2^m 5^n$ , where m, n are non negative integers. Then $x$ has a decimal expansion which terminates.**

**Theorem 1.7: : Let $x = \frac{p}{q}$, q $\neq$ 0 to be a rational number, such that the prime factorization of q is of the form $2^m 5^n$ , where m, n are non negative integers. Then $x$ has a decimal expansion which is non-terminating repeating**

**Example: 1 Express 140 as a product of its prime factor**

**Solution: 140 = 2 X 2 X 5 X 7**

**= $2^2$ X 5 X 7**

**Example: 2 SFind the HCF and LCM 91 and 26 by prime factorization.**

**Solution: 26 = 2 X 13**

**91 = 7 X 13**

**HCF = 13**

**LCM = 2 X 7 X 13 = 182**

**Example: 3 Find the HCF and LCM 12, 15 and 21 by prime factorization.**

**Solution: 12 = 2 x 2 x 3 = $2^2$ x 3**

**15 = 3 x 5**

**21 = 3 x 7**

**HCF = 3**

**LCM = $2^2$ x 3 x 5 x 7 = 420**

**Example: 4** **Given that HCF (306, 657) = 9, find LCM (306, 657).**

**Solution:** $\text{HCF}(306, 657) = 9$

We know that, $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$\text{LCM} = 22338$

**Example: 5** **Check whether $6^n$ can end with the digit 0 for any natural number *n*.**

**Solution:** If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also

be divisible by 2 and 5 as 10 = 2 x 5

Prime factorization of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorization of $6^n$.

Hence, for any value of *n*, $6^n$ will not be divisible by 5.

Therefore, $6^n$ cannot end with the digit 0 for any natural number *n*.

**Example: 6** **Explain why 7 x 11 x 13 + 13 and 7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 are composite numbers.**

**Solution:** Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, where as composite numbers have factors other than 1 and itself.

It can be observed that

7 x 11 x 13 + 13 = 13 x (7 x 11 + 1) = 13 x (77 + 1)

=13 x 78

=13 x 13 x 6

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 = 5 x (7 x 6 x 4 x 3 x 2 x 1 + 1)

=5 x (1008 + 1)

=5 x 1009

1009 can not be factorized further.

Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

**Example: 7**: **Prove that $\sqrt{5}$ is irrational.**

**Answer :** Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers $a$, $b$ ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

Let $a$ and $b$ have a common factor other than 1. Then we can divide them by the common factor, and assume that $a$ and $b$ are co-prime.

$$a = \sqrt{5}b$$
$$a^2 = 5b^2$$

Therefore, $a^2$ is divisible by 5 and it can be said that $a$ is divisible by 5.

Let $a = 5k$, where $k$ is an integer

$$(5k)^2 = 5b^2$$
$$b^2 = 5k^2$$

This means that $b^2$ is divisible by 5 and hence, $b$ is divisible by 5.

This implies that $a$ and $b$ have 5 as a common factor.

And this is a contradiction to the fact that $a$ and $b$ are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

**Example: 8** **Prove that $3+2\sqrt{5}$ is irrational.**

**Answer :**

Let $3+2\sqrt{5}$ is rational.

Therefore, we can find two integers $a$, $b$ ($b \neq 0$) such that

$$3+2\sqrt{5} = \frac{a}{b}$$
$$2\sqrt{5} = \frac{a}{b} - 3$$
$$\sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$$

**Since $a$ and $b$ are integers, $\frac{1}{2}\left(\frac{a}{b} - 3\right)$ will also be rational**

**And therefore, $\sqrt{5}$ is rational.**

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false.

Therefore, $3+2\sqrt{5}$ is irrational.

**Example: 9** **Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:**
**Answer :**

(i) $\frac{13}{3125}$

$3125 = 5^5$

The denominator is of the form $5^m$.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$8 = 2^3$

The denominator is of the form $2^m$.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

455=5×7×13

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

1600 = 26 × 52

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

**Example: 10** Using Euclid's division algorithm find the HCF of 225 and 135.

**Sol.** On applying the division lemma to 225 and 135

We get

$225 = 135 \times\times 1 + 90$

$90 = 45 \times 2 + 0$

Hence HCF(225,135) = 45

**Example: 11**

Use Euclid's division algorithm to find the HCF of 196 and 38220

**Sol.** 196 and 38220

We have 38220 > 196,

So, we apply the division lemma to 38220 and 196 to obtain

$38220 = 196 \times 195 + 0$

Since we get the remainder as zero, the process stops.

The divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

**Example :12**

**Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where *q* is some integer.**

**Sol.** Let $a$ be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times( 3q + 1) = 2k_1 + 1$, where $k_1$ is a positive integer

$6q + 3 = (6q + 2) + 1 = 2\,(3q + 1) + 1 = 2k_2 + 1$, where $k_2$ is an integer

$6q + 5 = (6q + 4) + 1 = 2\,(3q + 2) + 1 = 2k_3 + 1$, where $k_3$ is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where $k$ is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$, or $6q + 5$

**Example: 13**

**Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where *q* is some integer.**

**Sol.** Let $a$ be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times (3q + 1) = 2k_1 + 1$, where $k_1$ is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where $k_2$ is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where $k_3$ is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where $k$ is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$, or $6q + 5$

## WORK SHEET

1. Using prime factorization , find the HCF of
   (i) 405 and 2520
   (ii) 504 and 1188
   (iii) 960 and 1575
2. Using prime factorization, find the HCF and LCM of:
   (i) 36 and 84
   (ii) 23 and 31
   (iii) 96 and 404
   (iv) 144 and 198
   (v) 396 and 1080

In each case, verify that HCF X LCM = product of given number

3. Using prime factorization, find the HCF and LCM of:
   (i) 8, 9 and 25
   (ii) 12 ,15 and 21
   (iii) 17,23 and 29
   (iv) 24, 36 and 40
   (v) 30, 72 and 432
4. The HCF of two number is 23 and their LCM is 1449. If one of the number is 161, find the other.
5. The HCF of two number is 145 and their LCM is 2175. If one of the number is 725, find the other.
6. The HCF of two number is 18 and their product is 12960. Find their LCM.
7. State Euclid's division lemma.
8. State whether the given statement is true or false.
   (i) The sum of two rational is always rational.
   (ii) The product of two rational is always rational.

(iii) The sum of two irrational is always an irrational.

(iv) The product of two irrational is always an irrational.

(v) The sum of rational and an irrational is always irrational.

(vi) The product of rational and an irrational is always irrational.

CHAPTER NO. – 2

CHAPTER NAME – POLYNOMIALS

## KEY POINTS TO REMEMBER –

- **Geometrical meaning of the Zeroes of the Polynomial.**
- **Zeroes and coefficients of a Polynomial.**
- **Division Algorithm for polynomial.**

1. **MONOMIALS:** Algebraic expression with one term is known as Monomial.
2. **BINOMIAL:** Algebraic expression with two terms is called Binomial.
3. **TRINOMIAL:** Algebraic expression with three terms is called Trinomial.
4. **POLYNOMIALS**: All above mentioned **algebraic expressions are called Polynomials.**
5. **LINEAR POLYNOMIAL:** Polynomial with degree 1 is called Linear polynomial.
6. **QUADRATIC POLYNOMIAL:** Polynomial with degree 2 is called Quadratic polynomial.
7. **CUBIC POLYNOMIAL:** Polynomial with degree 3 is called Cubic Polynomial.
8. **BIQUADRATIC POLYNOMIAL: :** Polynomial with degree 4 is called bi-quadratic Polynomial.

## STANDARD FORM OF QUADRATIC POLYNOMIAL

- **A quadratic polynomial in x with real coefficients is the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.**
- **The zeros of a polynomial p(x) are precisely the x coordinates of the points where the graph of**
**y = p(x) intersect the x axis i.e. x = a is a zero of polynomial P (x) = 0.**
- **A polynomial can have at most the same number of zeros as the degree of polynomial.**
- **For Quadratic polynomial : $ax^2 + bx + c, \quad a \neq 0$**

  **Sum of zeroes = $\frac{-b}{a}$ and product of zeroes = $\frac{c}{a}$**

- **For cubic polynomials: $ax^3 + bx^2 + cx + d$ , if $\alpha, \beta, \gamma$ are the zeroes of the polynomial.**

**Then** $\alpha + \beta + \gamma = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

$\alpha x \beta x \gamma = = \frac{d}{a}$

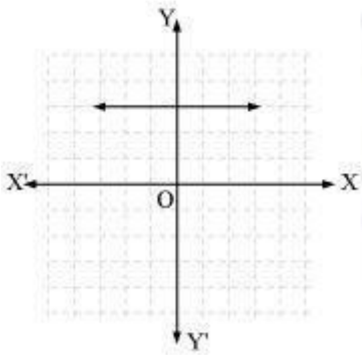
# CHAPTER - 2

## Polynomials

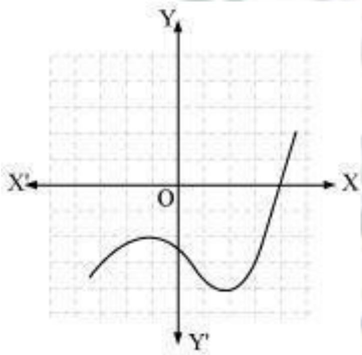
**Question 1:**

**The graphs of y = p(x) are given in following figure, for some polynomials p(x). Find the number of zeroes of p(x), in each case.**

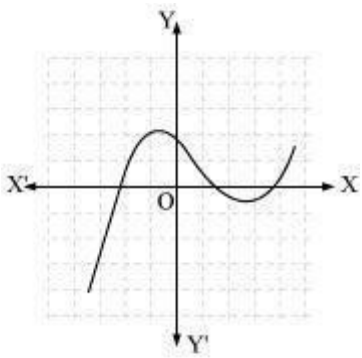
(i)



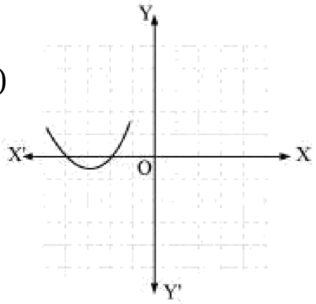

(ii)



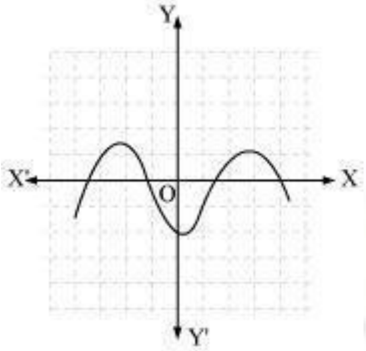

(iii)



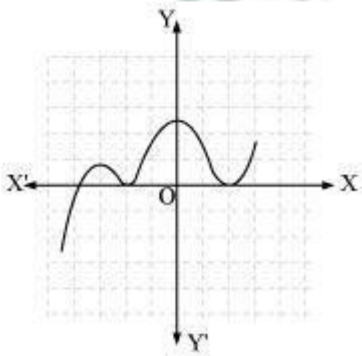
(iv)




(v)




(v)




**Answer:**

The number of zeroes is 0 as the graph does not cut the x-axis at any point.

The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

The number of zeroes is 4 as the graph intersects the x-axis at 4 points.

The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

**Question : 2**

**Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.**

$(\text{i})x^2-2x-8\ \ (\text{ii})4s^2-4s+1\ \ (\text{iii})6x^2-3-7x$

**Answer:**

(i) $x^2-2x-8=(x-4)(x+2)$

The value of $x^2-2x-8$ is zero when x − 4 = 0 or x + 2 = 0, i.e., when x = 4 or x = −2

Therefore, the zeroes of $x^2-2x-8$ are 4 and −2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of }x)}{\text{Coefficient of }x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of }x^2}$

(ii) $4s^2-4s+1=(2s-1)^2$

The value of $4s^2 - 4s + 1$ is zero when 2s − 1 = 0, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are s = ½, ½

.

Sum of zeroes = ½ + ½ = 2/2 =1

Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$

(iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x+1)(2x-3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e., $x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

**Question :3**
**Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.**

(i) $\frac{1}{4}, -1$

**Let the polynomial be $ax^2 + bx + c$, and its zeroes be $\alpha$ and $\beta$.**

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$
$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$
$$\text{If } a = 4, \text{ then } b = -1, \ c = -4$$

**Therefore, the quadratic polynomial is $4x^2 - x - 4$.**

(ii) $\sqrt{2}, \frac{1}{3}$

**Let the polynomial be $ax^2 + bx + c$, and its zeroes be $\alpha$ and $\beta$ .**

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$

**Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}\,x + 1$.**

CHAP – 2

# WORK SHEET

SUB: MATHS

## Answer the questions

1) If α and β are the zeros of quadratic polynomial $x^2 + px + 2q$, find the value of $\alpha^2 + \beta^2$.
2) If a and b are the zeros of quadratic polynomial $x^2 + 2px + q$, find the value of 1/a + 1/b.
3) If α and β are the zeros of quadratic polynomial x2 + 3x - 4, find the value of α3 + β3.
4) Find the zeros of the polynomial f(x) = x3 - 12x2 + 47x - 60, if it is given that sum of its two zeros is 9.
5) Find the quadratic polynomial such that sum of its zeros is 10 and difference between zeros is 8.
6) Find a quadratic polynomial whose zeros are reciprocals of the zeros of the polynomial x2 + 7x + 12.
7) If two zeros of polynomial x3 + bx2 + cx + d are 3+√3 and 3-√3, find its third zero.
8) If α and β are the zeros of polynomial x2 - 6x + k, such that α2 + β2 = 20. Find the value of k.
9) If α and β are the zeros of quadratic polynomial x2 - 4x - 5, find the value of 1/α3 + 1/β3.